Electrical Drives

3. Dynamics of Electrical drives

- A motor generally drives a load (machine) through some transmission system.
- While motor always rotates, the load may rotate or undergo translational motion, or both simultaneously.
- Load speed may be different from that of the motor.
- Representation of motor-load system may seem the following.

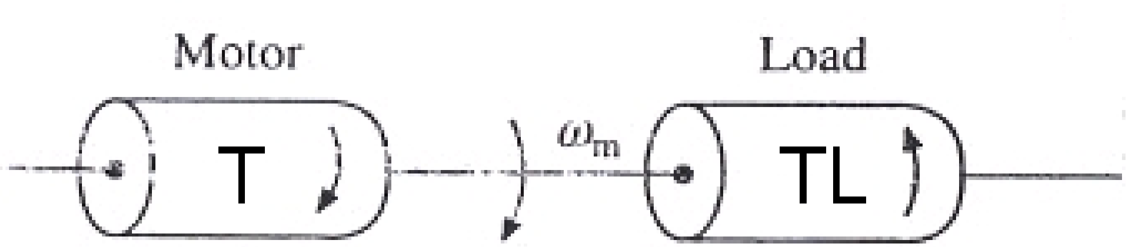


Fig. 3.1. Motor and load torque representation

- **Motor-load system can be described by the following fundamental torque equation (Equation of motion).**

$$T - T_L = \frac{d}{dt}\left(J\omega_m\right) = J\,\frac{d\omega_m}{dt} + \omega_m\,\frac{dJ}{dt} \qquad \text{---- } 3.1$$

For drives with constant inertia, (dJ/dt) = 0; thus,

$$T = T_L + J\frac{d\omega_m}{dt} \qquad \text{------------------} \qquad 3.2$$

Where, T = developed motor torque

T_L = Load torque referred to motor shaft.

J = polar moment of inertia of motor load system referred to motor shaft

ω_m = angular velocity of motor shaft

This equation shows that, torque developed by the motor is Counter balanced by a load torque T_L and a dynamic torque J(dω/dt).

- **Torque component $J(d\omega/dt)$ is called the dynamic torque because it appears during the transient operation.**

Analysis of the fundamental torque equation shows that;

- When $T > T_L$, $d\omega/dt > 0$. i.e. the drive undergo acceleration.
- When $T < T_L$, $d\omega/dt < 0$. i.e. the drive will undergo deceleration.
Deceleration occurs also at negative values of motor torque (i.e. during braking operation).
- When $T = T_L$, $d\omega/dt = 0$, i.e the drive will run at a steady state speed.

Speed –torque conventions and multi-quadrant operations.

- For considerations of multi-quadrant operation of drives, let us see suitable conventions about the signs of torques and speed.

- Motor speed is considered positive when rotating in the forward direction. For drives which operate only in one direction, forward speed will be their normal speed.

- In loads involving up-and-down motions, the speed of motor which causes upward motion is considered forward motion.

- For reversible drives, forward speed is chosen arbitrarily. Then the opposite direction gives reverse speed.

- Motor torque is considered positive, when it produces acceleration (positive rate of change of speed) in forward direction.
- Positive load torque is opposite in direction to the positive motor torque.
- Motor torque is considered negative if it produces deceleration.

- A motor operates in two modes. Motoring and braking.

- In motoring, it converts electrical energy to mechanical energy, which supports its motion.
- In braking, it works as a generator; opposing motion.

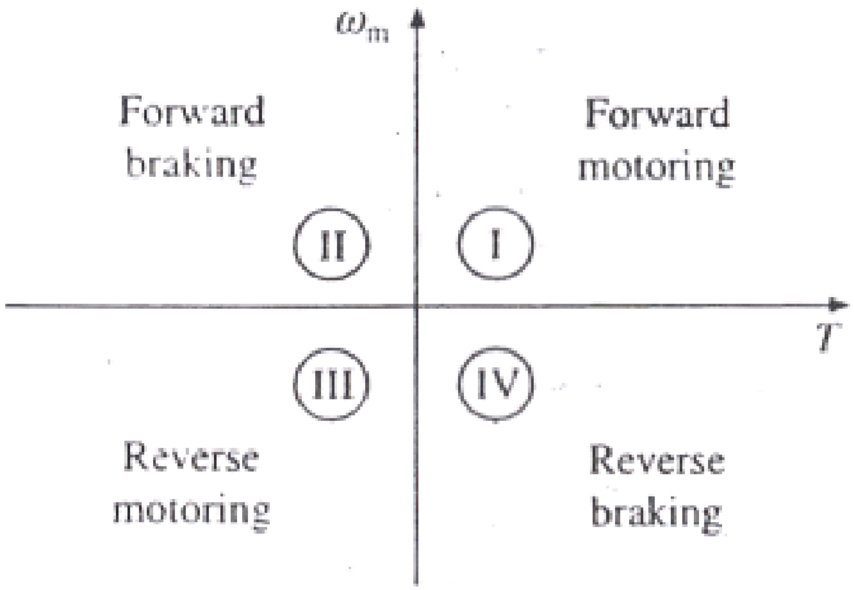
- A motor can provide motoring and braking operations for both forward and reverse directions.

- **The inertia or dynamic torque appears when the speed changes from one value to another. If the drive is undergoing acceleration, this torque opposes drive motion.**

- **If the drive is being braked, supports motion. The inertia torque both in magnitude and in sign, is determined as the algebraic difference between the motor torque and the load torque. In general the torque equation is written as;**

$$\pm T \mp T_L = J \frac{d\omega}{dt} \qquad \text{----------------------} \quad 3.3$$

Selection of the sign to be placed before each of the torque depends on the operating conditions and on the nature of Load torque.



Power developed by a motor is given by the product of speed and torque.

In quadrant I, power Developed is positive; machine works as a motor supplying Mechanical energy.

In quadrant II, power is negative. Hence the machine works under Braking opposing the motion.

Fig. 3.2. Multi-quadrant operation of drives

Example

- A motor drives two loads. One has rotational motion. It is coupled to the motor through a reduction gear with i= = 0.1 and efficiency of 90%. The load has a moment of inertia of 10 Kg-m^2 and a torque of 10 N-m. Other load has translational motion and consists of 1000 Kg weight to be lifted up an uniform speed of 1.5 m/s. Coupling between this load and the motor has an efficiency of 85%. Motor has an inertia of 0.2 Kg-m^2 and runs at a constant speed of 1420 rpm. Determine the equivalent inertia referred to the motor shaft and power developed by the motor.

Given;

$J_0 = 0.2 kg\text{-}m^2$; $\eta 1 = 0.9$

$i_1 = 0.1$; $\eta'_1 = 0.85$

$J_1 = 10 kg\text{-}m^2$ $T = 10$ N-m

$V = 1.5$ m/s; $G = 1000$ kg

$\omega_m = (1420x\pi/30) = 148.7$ rad/sec

Sol

The total moment of inertia referred to the motor shaft

$$J = J_0 + i_1^{\,2} J_1 + M_1 \left(\frac{v_1}{\omega_m} \right)^2$$

$$J = 0.2 + (0.1)^2 \times 10 + 1000 \left(\frac{1.5}{148.7} \right)^2 = 0.4\, kg - m^2$$

$$T_L = \frac{i_1 T_{L1}}{\eta_1} + \frac{F_1}{\eta_1^{'}} \left(\frac{v_1}{\omega_m} \right)$$

$$T_L = \frac{0.1 \times 10}{0.9} + \frac{1000 \times 9.81}{0.85} \left(\frac{1.5}{148.7} \right) = 117.53\, N - m$$

Components of Load torques

Load torque T_l can be divided into the following components;

i) Friction torque T_F – Friction will be present at the motor shaft and also in various parts of the load. T_F is equivalent value of various friction torques referred to the motor shaft.

ii) Windage Torque T_w – It is a torque generated when a motor runs, which opposes the motion.

iii) Torque required to do the useful mechanical work T_L – Nature of this torque depends on particular application.

- It may be constant and independent of the speed;
- It may be some function of speed;
- It may depend on the position or path followed by the load;
- It may be time invariant or time variant;
- It may vary cyclically and its nature may also change with the load's mode of operation

- **Variation of friction torque with speed is shown in the following figure;**

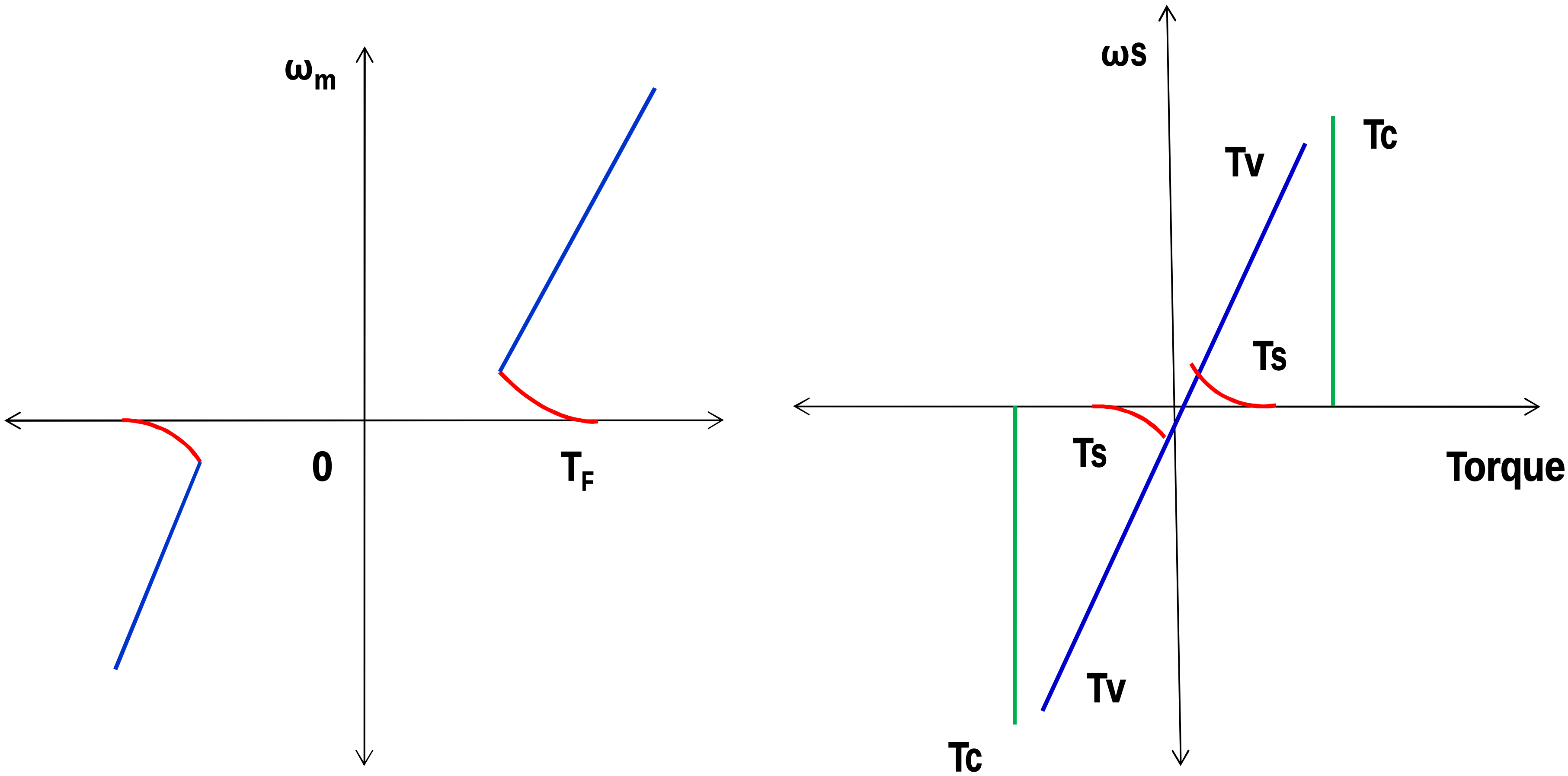


Fig. 3.5. Variation of Friction torque and its components

- As can be seen from the figure, the Friction torque value at stand still is much higher than its value slightly above zero speed.
- Friction at zero speed is called Stiction or static friction. In order for a drive to start, the motor torque should at least exceed Stiction.
- Friction torque can be resolved into three components (see the fig.)
  - Viscous friction torque (T_v) : - component which varies linearly with speed.

    $T_v = B\omega_m$; where B is viscous friction coefficient.
  - Coulomb friction (T_c) which is independent of speed.
  - Additional friction torque at standstill (Ts): - Ts is present only at stand still and is not taken into account in the dynamic analysis.

- **Windage friction torque (T_w): - which is proportional to the speed squared.**

$T_w = C\omega^2_m$; **where C is a constant**

- **From the above discussion, for finite speeds;**

$$T_l = T_L + B\omega_m + T_c + C\omega^2_m$$

where, T_l – instantaneous value of load torque referred to motor shaft.

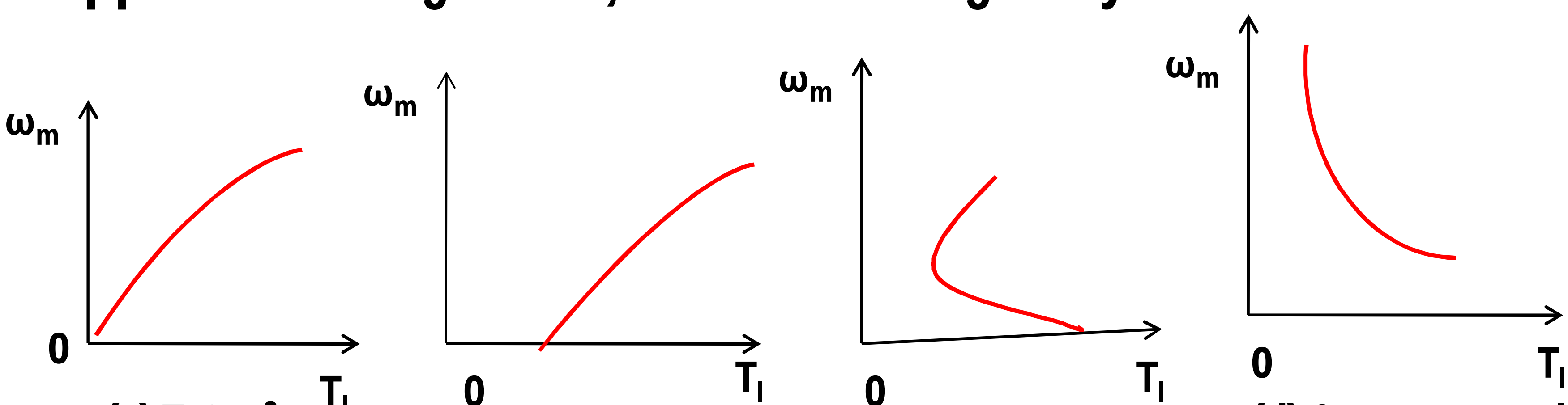
T_L – torque required to do the useful mechanical work.

. In many applications, ($T_c + C\omega^2_m$) is very small and can be neglected. Then the fundamental torque equation becomes;

$$T = J\frac{d\omega_m}{dt} + T_L + B\omega_m \quad \text{------------} \quad 3.9$$

Nature and classification of load torques

- As stated earlier, the nature of load torque depends on particular application.
- Fans, compressors, centrifugal pumps, ship-propellors, coilers, high speed hoists, traction, etc… are example of the case where load torque is proportional to speed squared. (fig. (a) and (b))
- In a high speed hoist, viscous friction and windage also have appreciable magnitude, in addition to gravity.



(a) $T_l \propto \omega_m^2$

(b) High speed hoist

(c) Traction load

(d) Const. power load (coiler machine)

fig. 3.6. Steady state load torque speed curves

- Figure (c) shows the traction load to be function of only speed, assuming a leveled ground. In actual practice the train has to negotiate upward and downward slopes. Consequently, a torque due to gravity, which varies with position is also present. Furthermore, when a train takes a turn, the friction force on wheels changes substantially. Thus traction is an example where the load torque also depends on position or path followed.

Time and energy loss in Transient operations

- **Starting, braking, speed change and speed reversal are transient operations.**
- **Time taken and energy dissipation in the motor during transient operations can be evaluated by solving the following equation along with motor circuit equations.**

$$T = J\frac{d\omega_m}{dt} + T_L + B\omega_m \qquad \text{-------------} \quad \text{Eq. 3.9.}$$

- **For any of the above mentioned transients, final speed is an equilibrium speed.**
- **Theoretically, transients are over in infinite time , which is not so in practice. However, transient operation is considered to be over when 95% change in speed has taken place.**

- **For example, when speed changes from ω_{m1} to equilibrium speed ω_{me}, time taken for the speed to change from ω_{m1} to $[\omega_{m1} + 0.95(\omega_{me} - \omega_{m1})]$ considered to be equal to the transient time. From the following equation (eq. 3.2)**

$$T = T_L + J\frac{d\omega_m}{dt}$$

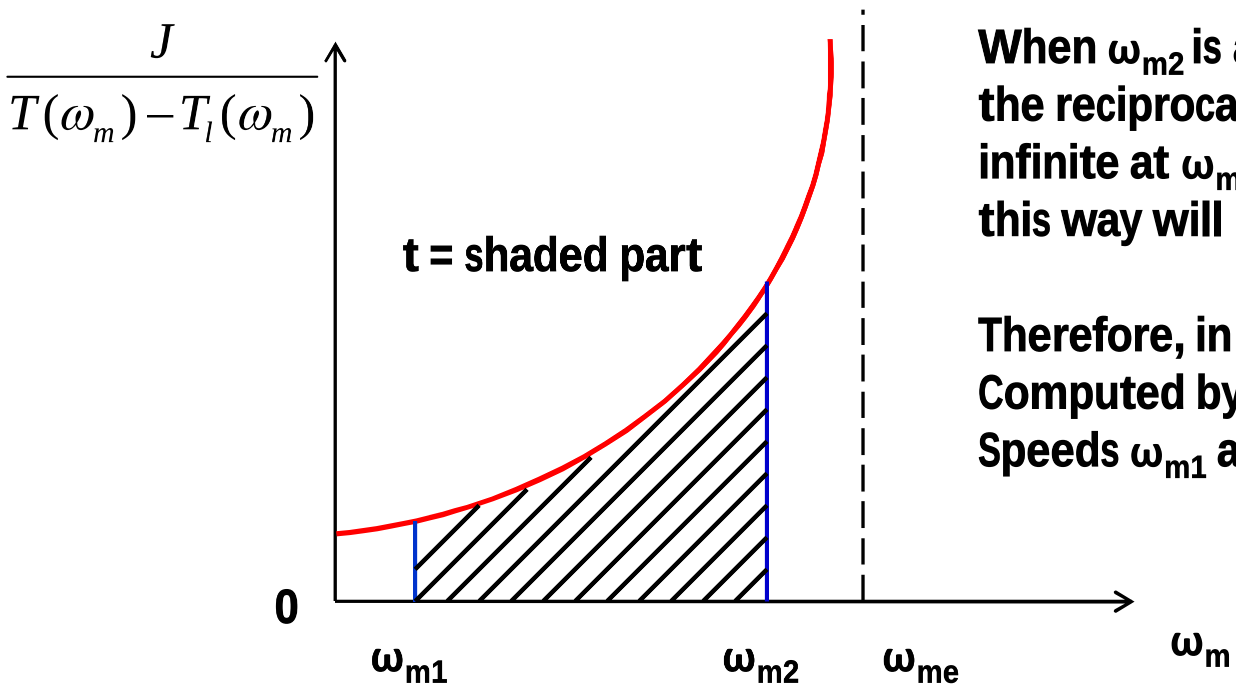
$$dt = \frac{J d\omega}{T(\omega_m) - T_l(\omega_m)} \qquad \text{----------------} \qquad 3.10$$

- **If $T(\omega_m)$ and $T_l(\omega_m)$ are constants, then solution of equation 3.10. will be;**

$$t = J\,\frac{\omega_{m2} - \omega_{m1}}{T + T_l} \qquad \text{--------------} \qquad 3.11$$

- **Since $T(\omega_m)$ and $T_l(\omega_m)$ are functions of speed, the solution could be obtained graphically.**

$$t = J\int_{\omega_{m1}}^{\omega_{m2}} \frac{d\omega_m}{T(\omega_m) - T_l(\omega_m)} \quad \text{-------------} \quad 3.12$$



When ω_{m2} is an equilibrium speed ω_{me}, then the reciprocal of integration will become infinite at ω_{me}. Consequently time evaluated this way will be infinite.

Therefore, in this case, transient time is Computed by measuring the area between Speeds ω_{m1} and $\omega_{m1} + 0.95(\omega_{m2} - \omega_{m1})$

Fig. 3.7. Calculation of time during a transient operation

- **Energy dissipated in a motor winding during a transient operation is given by;**

$$E = \int_0^t i^2 R dt$$

Where, R is the motor winding resistance
i is current flowing through the winding

STEADY STATE STABILITY OF A DRIVE SYSTEM

- **Equilibrium speed of a motor- load system is obtained when motor torque equals the load torque .**

- **a drive will operate in steady state at this speed, provided it is the speed of stable equilibrium.**

- **In most drives the electrical time constant of the motor is negligible compared to its mechanical time constant. Therefore, during transient operations, motor can be assumed to be in electrical equilibrium implying that steady-state speed-torque curves are also applicable to the transient operations.**

Let us see the following example.

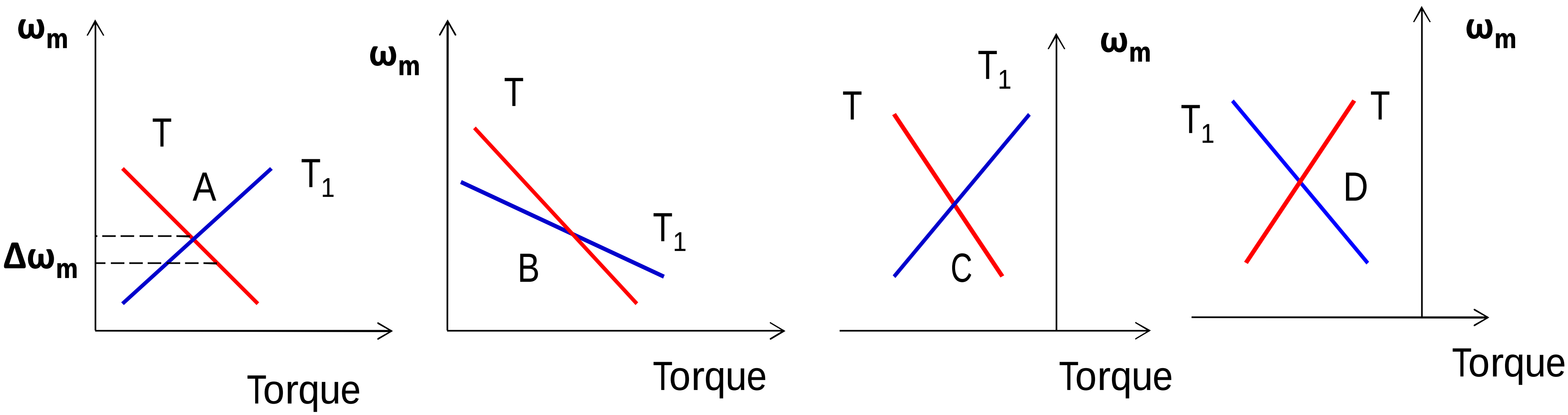


fig. 3.8. **Points A and C are stable and B and D are unstable.**

- **The equilibrium point (example point A) will be termed as stable when the operation will be restored to its initial position after a small departure from it due to a disturbance in the motor or load.**

- Let the disturbance causes a reduction of $\Delta\omega_m$ in speed. At new speed, motor torque is greater than the load torque, consequently, motor will accelerate and operation will be restored to A.

- Similarly, an increase of $\Delta\omega_m$ in speed caused by a disturbance will make load torque greater than the motor torque, resulting into deceleration and restoration of operation to point A. Hence, the drive is steady state stable at point A.

- Let us now examine equilibrium point B which is obtained when the same motor drives another load.
- A decrease in speed causes the load torque to become greater than the motor torque, drive decelerates and operating point moves away from B.
- Similarly, an increase in speed will make motor torque greater than the load torque which will move the operating point away from B. Thus, B is an unstable point of equilibrium.

For points C and D try to analyze yourself.

- The Above discussion suggests that an equilibrium point will be stable when an increase in speed causes load torque to exceed the motor torque; i.e. when at equilibrium point following condition is satisfied.

$$\frac{dT_l}{d\omega_m} > \frac{dT}{d\omega_m} \qquad \text{------------------} \qquad 3.13$$